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Bayesian and Non-Bayesian Estimation Methods for Parameters of Some New Lifetime Models

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DISSERTATION

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بسم الله الرحمن الرحيم

﴿ وَمَا تَوْفِيقِيٓ إِلَّا بِٱللَّهِۚ عَلَيْهِ تَوَكَّلْتُ وَإِلَيْهِ أُنِيبُ ﴾

صدق الله العظيم

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Summary

Statistical distributions have a great importance in the scientific research which are commonly applied to describe the real phenomena. Thus, several statistical distributions have been extensively used in data analysis in several areas of research such as medical, engineering, reliability, etc. However, in many situations, the classical distributions are not appropriate for describing the data and the real phenomena, so many statisticians have introduced new families of distributions by adding a shape parameter(s) to these distributions to deal with this problem. Thus, in this study, three distributions named “Beta Exponentiated Kumaraswamy distribution”, “Beta Exponentiated Generalized Extended Pareto distribution”, and “Beta Exponentiated Inverse Rayleigh distribution” are presented. Some statistical properties and Maximum Likelihood estimation are obtained. Bayesian estimation is studied for each distribution for 3 types of priors that included informative and non-informative prior. Applied studies for Beta Generalized Inverted Kumaraswamy distribution, Beta Exponentiated Generalized Extended Pareto distribution, and Beta Exponentiated Inverse Rayleigh distribution are provided using Mathematica Package 12. we used the Monte Carlo simulation to study the behavior of the parameter’s estimator in different samples size for different priors, and we used the R language to conduct the Monte Carlo simulation study.

This dissertation consists of five chapters that deal with generating some new lifetime models, obtained its properties and estimated the parameters of these models with classical method “Maximum Likelihood estimation” and Bayesian estimation using Markov Chain Monte Carlo simulation.

Chapter 1: Introduction

This chapter is divided into nine subsections:

- **1.1** provides an overview of the
- **1.2** some definitions needed to recognize that will be used in this study.
- **1.3** Parameter estimation methods
- **1.4** Literature Review of Kumaraswamy distribution.
- **1.5** Literature Review of Pareto distribution.
- **1.6** Literature Review of Inverse Rayleigh distribution.
- **1.7** Literature Review of Class of Beta Generalized distribution.
- **1.8** Literature Review of Parameter estimation.
- **1.9** the main objectives of the current dissertation.

Chapter 2: Beta Exponentiated Kumaraswamy Distribution

This chapter is divided into seven subsections:

- **2.1** presents the generation of BEKw distribution.
- **2.2** introduces an Expansion for pdf and cdf of BEKw distribution.
- **2.3** discusses some general statistical properties of the BEKw distribution.
- **2.4** presents the estimation of the parameters for BEKw distribution using Maximum Likelihood method.
- **2.5** gives Bayesian estimation of the parameters for BEKw distribution with both two informative prior and non-informative prior.
- **2.6** summarizes MCMC for BEKw distribution using Metropolis Hasting algorithm.

- **2.7** introduces a Monto Carlo Simulation for BEKw model and numerical study applies to a real data set to examine the performance of our model.

Chapter 3: Beta Exponentiated Generalized Extended Pareto Distribution.

This chapter is divided into seven subsections:

- **3.1** presents the generation of BEGEP distribution.
- **3.2** introduces an Expansion for pdf and cdf of BEGEP distribution.
- **3.3** discusses some general statistical properties of the BEGEP distribution.
- **3.4** presents the estimation of the parameters for BEGEP distribution using Maximum Likelihood method.
- **3.5** gives Bayesian estimation of the parameters for BEGEP distribution with both two informative prior and non-informative prior.
- **3.6** summarizes MCMC for BEGEP distribution using Metropolis Hasting algorithm.
- **3.7** introduces a Monto Carlo Simulation for BEGEP model and numerical study applies to a real data set to examine the performance of our model.

Chapter 4: Beta Exponentiated Inverse Rayleigh Distribution

This chapter is divided into seven subsections:

- **4.1** presents the generation of BEIRey distribution.
- **4.2** introduces an Expansion for pdf and cdf of BEIRey distribution.
- **4.3** discusses some general statistical properties of the BEIRey distribution.

- **4.4** presents the estimation of the parameters for BEIRey distribution using Maximum Likelihood method.
- **4.5** gives Bayesian estimation of the parameters for BEIRey distribution with both two informative prior and non-informative prior.
- **4.6** summarizes MCMC for BEIRey distribution using Metropolis Hasting algorithm.
- **4.7** introduces a Monto Carlo Simulation for BEIRey model and numerical study applies to a real data set to examine the performance of our model.

Chapter 5: Conclusions

This chapter is divided into two subsections:

- **5.1** we summarize this dissertation.
- **5.2** general recommendations and future work were proposed.

Glossary of Notations

Notation	Definition
ABRey	Area Biased Rayleigh
APP	Alpha Power Pareto
BEGEP	Beta Exponentiated Generalized Extended Pareto
BEIRey	Beta Exponentiated Inverse Rayleigh
BEKw	Beta Exponentiated Kumaraswamy
C_1	Case 1
C_2	Case2
C_3	Case 3
D	Distribution
EGEP	Exponentiated Generalized Extended Pareto
EIRey	Exponentiated Inverse Rayleigh
EKw	Exponentiated Kumaraswamy
EP	Extended Pareto
ExP	Exponentiated Pareto
GIKw	Generalized Inverted Kumaraswamy
Irey	Inverse Rayleigh
Kw	Kumaraswamy
LEKw	Log Exponentiated Kumaraswamy
MCMC	Markov Chain Moto Carlo

MH	Metropolis hasting
MSE	Mean Square Error
MLE	Maximum Likelihood Estimate
OLRey	Odd Lindely Rayleigh
P	Pareto
Rey	Rayleigh
SBKw	Sized Biased Kumaraswamy
SEL	Square error loss function
TIRey	Transmuted Inverse Rayleigh
TKw	Transmuted Kumaraswamy
TP	Transmuted Pareto

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